

Predicting the Reservoir Pressure Effects of Injection and Production Wells Using a Solution of The Diffusivity Equation *(Oil Field Units)*



EPA, Region 6
Dallas, Texas

José Eduardo Torres
Petroleum Engineer
Chemical Engineer

THE DIFFUSIVITY EQUATION AS PRESENTED BY CRAFT AND HAWKINS

$$p = p_e + \frac{q\mu B_o}{14.16 kh} Ei\left[\frac{-r^2}{4\eta t}\right]$$

The Diffusivity Factor →

$$\eta = \frac{6.32 k}{\mu c_o \phi_{HC}} =$$

Computation of the Exponential Integral Ei

$$Ei(-x) = \ln x + 0.5772 - x + \frac{x^2}{2 \times 2!} - \frac{x^3}{3 \times 3!} + \frac{x^4}{4 \times 4!} - \dots - \frac{x^n}{n \times n!}$$

The equation may be used to find the pressure drop ($p_e - p$) which will have occurred at any radius about the well after flowing at a rate q for t days. For example, in a reservoir where $\mu_o = 0.72$ cp; $B_o = 1.475$ bbl/STB; $k = 100$ md; $h = 15$ ft; $c_e = 15 \times 10^{-6}$ psi $^{-1}$; $\phi_{HC} = 23.4$ per cent hydrocarbon porosity; $p_e = 3000$ psia; after a well is produced at 200 STB/day for 10 days the pressure at a radius of 1000 ft will be

$$p = 3000 + \frac{200 \times 0.72 \times 1.475}{14.16 \times 0.10 \times 15} Ei \left[\frac{-(1000^2)}{4 \times 25 \times 10^4 \times 10} \right]$$

where

$$\eta = \frac{6.32 k}{\mu c_e \phi_{HC}} = \frac{6.32 \times 0.10}{0.72 \times 15 \times 10^{-6} \times 0.234} = 25 \times 10^4$$

Then

$$p = 3000 + 10.0 Ei(-0.10)$$

From Fig. 6.38, $Ei(-0.10) = -1.82$. Therefore

$$p = 3000 + 10.0 \times (-1.82) = 2981.8 \text{ psia}$$

Figure 6.39 shows this pressure plotted on the 10-day curve, and, in addition, curves showing the pressure distributions at 0.1, 1.0, 10, and 100 days for the same flow conditions.

Craft and Hawkins
Example on the
Use of the
Diffusivity Equation

Reservoir Pressure
Distribution about
a Production Well
(Drawdown)

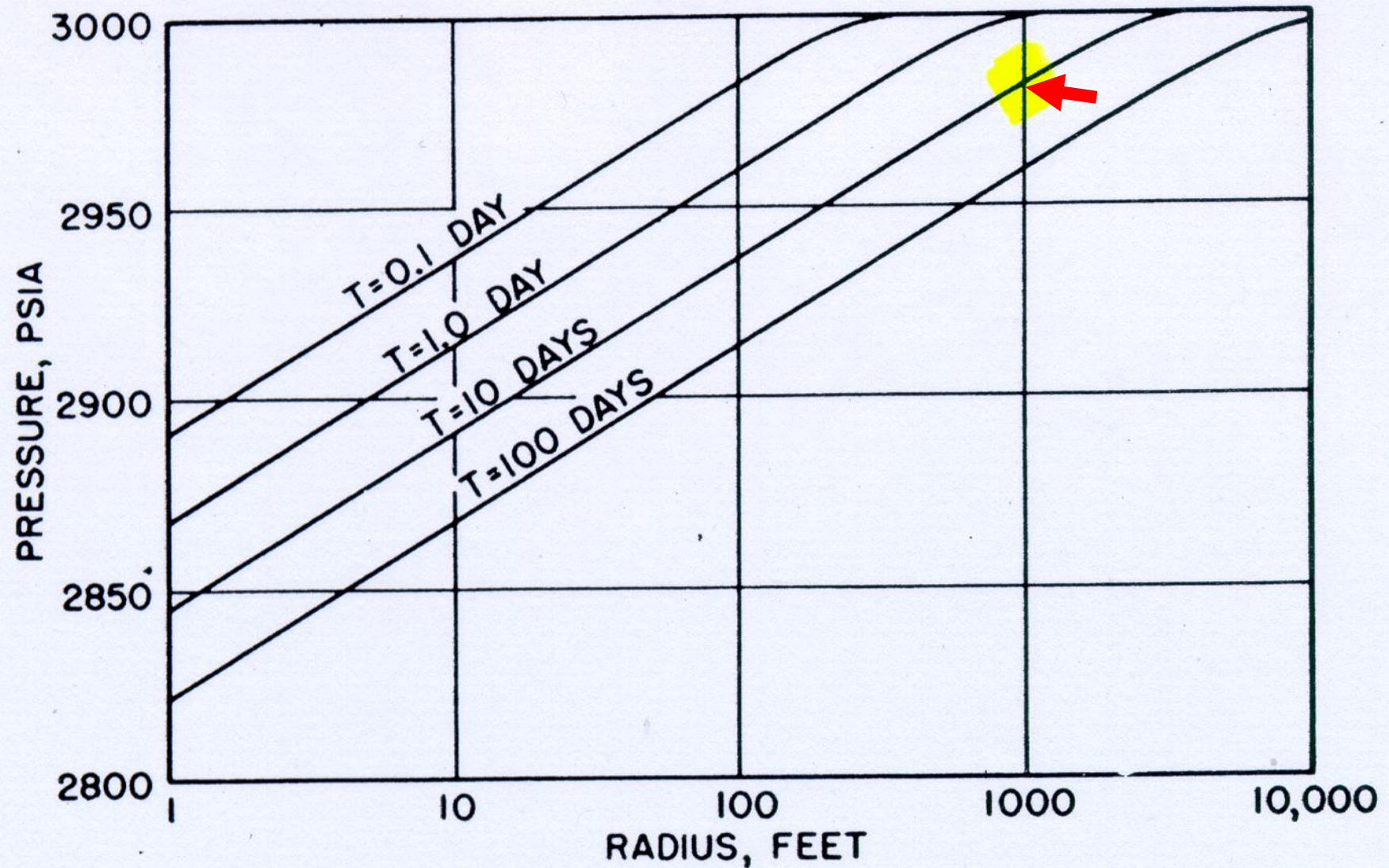


Fig. 6.39. Pressure distribution about a well at four time periods after start of production. $q = 200$ STB/day; $\mu_o = 0.72$ cp; $B_o = 1.475$ bbl/STB; $k = 100$ md; $h = 15$ ft; $c_e = 15 \times 10^{-6}$ psi $^{-1}$; $\phi_{HC} = 23.4$ per cent; $p_e = 3000$ psia.

The Craft and Hawkins
Example Computations
implemented with an
Excel Computer Program

FLOW OF FLUIDS IN POROUS MEDIA			
Reservoir Pressure Effects Computations			
Production Rate (STBb/Day)	200	Compute Diffusivity Factor	
Time of Operation (Days)	10	Numerator	0.632
Initial Reservoir Pressure (psia)	3000	Denominator	2.53E-06
Injected Fluid Viscosity (cp)	0.72	Diffusivity Factor = 2.50E+05	
Formation Volume Factor (ResBb/STBbl)	1.475	Compute "x"	
Formation Porosity (Percent)	23.4	Numerator	1000000
Formation Permeability (md)	100	Denominator	1.00E+07
Formation Interval Thickness (Ft)	15	x = 1.00E-01	
Formation Compressibility (1/psi)	1.50E-05	Compute Ei(-x)	
Specified Radius (Ft)	1000	0.00249842	
Computed Pressure Change @ Specified Radius (psia)	-18.23	5.5503E-05	
Resulting Reservoir Pressure @ Specified Radius (psia)	2981.8	1.0403E-06	
		1.664E-08	
		Ei(-x) -1.82	
		Compute Reservoir Pressure Change @ Specified Radius	
		Numerator	212.4
		Denominator	21.24
		Pressure Change = -18.23	

Reference:

